LEVEL II

INSTITUTE OF DECISION SCIENCE
FOR BUSINESS & PUBLIC POLICY

A DECREASING FAILURE RATE,

MIXED EXPONENTIAL MODEL

APPLIED TO RELIABILITY*

BY

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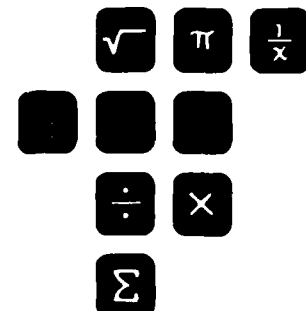D

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Decreasing failure rates for electronic equipment used on the Polaris, Posiedon and Trident missile systems have been observed. The mixed exponential distribution has been shown to fit the life data for the electronic equipment on these systems. This paper discusses some of the estimation problems which occur with the decreasing failure rate mixed exponential distribution when the test data is censored and only a few failures are observed. For these cases sufficient conditions are obtained that maximum likelihood estimators of the shape and scale parameters for the distribution exist. Actual data, obtained from the testing

20.
of missile electronic packages, are provided to illustrate these concepts and verify the applicability and usefulness of the techniques described.

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1. Introduction

There have been only a few parametric models extensively examined for application to reliability; these include the exponential distribution of Epstein-Sobel [6], the Weibull distribution [14], and the fatigue model of Birnbaum-Saunders [4]. The one most widely utilized for electronic components has been the exponential model, not only because of its simple and intuitive properties but also because of the extent of the estimation and sampling procedures which have been developed from the theory.

One of the early discoveries was that mixtures of exponentially distributed random variables have a decreasing failure rate, see [11]. Thus any two groups of components with constant, but different, failure rates would, if mixed and sampled at random, exhibit a decreasing failure rate. As a consequence, the family of life lengths with decreasing failure rate certainly arises in practice and particular subsets of this family could be of great utility for specific applications, see e.g. Cozzolino [5]. We examine one such model with shape and scale parameters, call them α and β respectively, which is based upon a particular mixture of exponential distributions. This family was introduced by Afanas'ev [1] and later by Lomax [10] as a generalization of a Pareto distribution. Section 3 compares this mixed exponential distribution to the exponential distribution using data from Poseidon flight control packages.

Kulldorff and Vännman [9] and Vännman [13] have studied a variant of this mixed exponential model containing a location parameter. They

obtained a best linear unbiased estimate of the scale parameter
assuming that the shape parameter, call it α, was known and in a
region restricted so that both the mean and the variance exist, namely
$\alpha > 2$. When this restriction of $\alpha > 2$ cannot be met an estimate based
on a few order statistics, which are optimally spaced, is claimed
to be an asymptotically best linear unbiased estimate and tables
of the weights as functions of the number of spacings are provided. In
all cases, the shape parameter was assumed known and the sample was
either complete or type II censored. It is contended that BLUE
estimates of the shape parameter are not attainable.

Harris and Singpurwalla [7] examined the method of moments as an
estimation procedure for this same model but again with the shape
parameter restricted to $\alpha > 2$ and with a complete sample.

In both papers [9] and [7], it is stated that maximum likelihood
estimates are difficult to obtain. In a later paper Harris and
Singpurwalla [8] exhibit the maximum likelihood equations for complete
samples.

In this paper the maximum likelihood estimates for both the shape
and scale parameters are obtained, jointly and separately, with simple
sufficient conditions given for their existence. These estimates are
derived for censored data (and a fortori for complete samples) even with
a paucity of failure observations, namely one.

The existence conditions obtained here for the maximum likelihood
estimates apply even to the case where the variance and possibly the
mean do not exist: $0 < \alpha < 2$. Moreover, the estimates of the shape

parameter α which have been obtained from actual data indicate that
this region $0 < \alpha < 2$ is important because all the estimates obtained
of α have been less than unity.

2. The Model

We postulate that the underlying process which determines the length
of life of the component under consideration is the following: The
quality of construction determines a level of resistance to stress which
the component can tolerate. The service environment provides shocks
of varying magnitude to the component, and failure takes place when for
the first time the stress from an environmentally induced shock exceeds
the strength of the component.

If the time between shocks of any magnitude is exponentially
distributed with a mean depending upon that magnitude then the life
length of each component will be exponentially distributed with a failure
rate which is determined by the quality of assembly. It follows that
each component has a constant failure rate but that the variability in
manufacture and inspection techniques forces some components to be
extremely good while a few others are bad and most are in between.

Let X_λ be the life length of a component in such a service
environment, with a constant failure rate λ which is unknown. The
variability of manufacture determines various percentages of the λ-values
and this variability can be described by some distribution, say G.

Let T be the life length of one of the components which is selected at random from the population of manufactured components. We denote the reliability of this component by R and we have

$$R(t) = P\left[T > t\right] \qquad \text{for} \quad t > 0 .$$

Let Λ be the random variable which has distribution G. We can write

$$R(t) = E_\Lambda P\left[X_\lambda > t \mid \Lambda = \lambda\right] = \int_0^\infty e^{-\lambda t} dG(\lambda) . \qquad (1)$$

Because of having a form which can fit a wide variety of practical situations when both scale and shape parameters are disposable, it is assumed that G is a gamma distribution, i.e., for some $\alpha > 0$, $\beta > 0$.

$$g(\lambda) = \frac{\lambda^{\alpha-1} e^{-\lambda/\beta}}{\Gamma(\alpha) \beta^\alpha} \qquad \text{for } \lambda > 0 .$$

That this assumption is robust, even when mixing as few as five equally weighted λ's, has been shown by recent work of Sunjata in an unpublished thesis [12]. It follows from equation (1) that the reliability function is

$$R(t) = \frac{1}{(1+t\beta)^\alpha} = e^{-\alpha \ln(1+t\beta)} . \qquad (2)$$

The failure rate, hazard rate, can be shown to be

$$q(t) = \frac{\alpha\beta}{1+t\beta} , \qquad (3)$$

which is a decreasing function of $t > 0$.

Maximum likelihood estimates for α, β and hence $R(t)$ and $q(t)$ are given in Section 5.

3. A Comparison of the Mixed Exponential with Exponential Using Real Data

Data has been accumulating for years in the assessment of the reliability of electronic equipment for which there was no adequate statistical model. The following difficulties were recognized by practitioners: 1. The assumption of constant or increasing failure rate seemed to be incorrect. 2. However, the design of this electronic equipment indicated that individual items should exhibit a constant failure rate. A mixed exponential life distribution accounts for both the design knowledge and the observed life lengths. Maximum likelihood procedures allow for joint estimation of the parameters of this distribution in the most commonly encountered situation where complete data is not available.

We now give some actual data sets from two different lots of Poseidon flight control electronic packages which illustrate these points. Each package has recorded, in minutes, either a failure time or an alive time. An alive time is sometimes called a "run-out" and is the time the life test was terminated with the package still functioning.

First Data Set

Failure times: 1, 8, 10

Alive times: 59, 72, 76, 113, 117, 124, 145, 149, 153, 182, 320.

Second Data Set

Failure times: 37, 53

Alive times: 60, 64, 66, 70, 72, 96, 123.

If we assume that the data are observations from an exponential distribution (constant failure rate λ) then using the total life statistic, we have the estimates of reliability given in the left hand side of the table. If we assume that the data are observations from the mixed exponential distribution of equation (2) then using estimation techniques derived subsequently in this paper we have the estimates for reliability given in the right hand side of the table.

time t in min.	Exponential estimate of reliability		Mixed exponential estimate of reliability	
	Set 1 $\hat{R}_1(t)$	Set 2 $\hat{R}_2(t)$	Set 1 $\hat{R}_1(t)$	Set 2 $\hat{R}_2(t)$
6	.988	.981	.915	.976
10	.980	.969	.896	.961
30	.943	.911	.855	.896
50	.906	.856	.836	.843
100	.821	--	.810	--
130	.774	--	.801	--
$\hat{\lambda}$:	.00017	.00312	$\hat{a}$: .0453	.420
			$\hat{\beta}$: 1.03	.01

Looking at the data from the two sets we would expect that at least for the first fifty minutes the reliability estimate for the second set of data would be <u>higher</u> then the reliability estimate for the first set of data, because in the first set 3 failure out of 14 trials have occurred in the first ten minutes while in the second set only 1 failure out of 9 trials has occurred in the first fifty minutes. However, under the

exponential assumption the reliability estimates for the first data set
are consistently higher. Note that the mixed exponential estimates
are more consistent with what the data show; that is, for at least the
first 50 minutes we expect the reliability estimate for the second set
of data to be higher than the reliability estimate for the first set
of data. Beyond this time, however, say at 100 minutes, the data
indicate that the reliability estimate from the first set of data should
be higher than the reliability estimate from the second set of data.
Using mixed exponential estimates this is the case.

A statistical test to determine whether the data require a constant
or decreasing failure rate was run on the data from Sets 1 and 2. For
data Set 1 we reject constant failure rate in favor of decreasing failure
rate at the .10 level. For data Set 2 we cannot reject the constant
failure rate assumption. In this case, however, the constant failure
rate estimates for reliability and the mixed exponential estimates for
reliability are close. For data Set 2 one should not estimate
reliability much beyond about 70 minutes since we do not have data to
support those estimates.

4. Residual Life Property of the Model

An important property of this model is that residual life on a
component is distributed as a mixed exponential. Thus a "burn-in"
test of a component will yield a residual life which is also in the same
family. This property seems to be shared only with the exponential among
common parametric families of life distributions.

The residual life T_h of a component is defined to be the life remaining after time h, given that the component is alive at time h. It can be shown that:

A burn-in for h units of time on a component with initial life determined by a mixed exponential distribution with parameters α and β will yield a residual life T_h and will be destributed as a mixed exponential with parameters α and $\dfrac{\beta}{1 + \beta h}$.

It follows that this life length model is "used better than new" or "new worse than used" in the sense that we have stochastic inequality between a new component and one that has been burned in, namely

$$T \overset{st}{\leq} T_h \qquad \text{for all} \quad h > 0.$$

An important consequence of this property is that one can calculate the value of the increased reliability attained by burn-in procedures as compared with the cost of conducting them. It has long been the practice to burn in electronic components based on intuitive ideas of "infant mortality" in order to provide reasonable assurance of having detected all defectively assembled units. This model, whenever it is applicable, makes possible an economic analysis. A variation of this result has been discussed in [3].
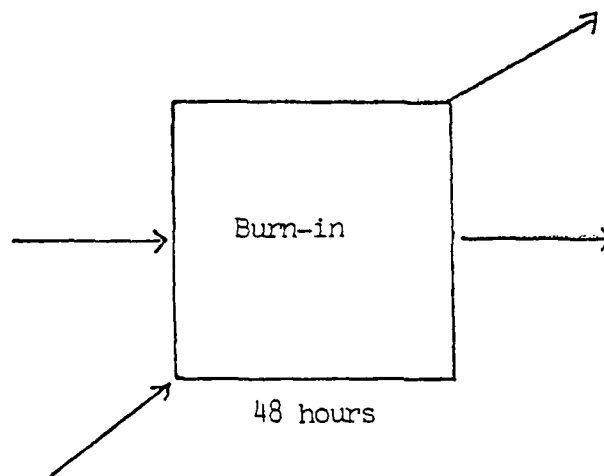
Example

As an example of the applicability of this property, consider test data from Trident flight control packages.

Assume that burn-in data is distributed as a mixed exponential with shape parameter α and scale parameter β. These parameters were estimated (formulas in Section 5) to be

$$\hat{\alpha} = .57$$
$$\hat{\beta} = .0104$$

$$\tilde{\beta}_f = \frac{\hat{\beta}}{1+(48)(60)\hat{\beta}}$$

Burn-in

48 hours

$$\hat{\alpha} = .57$$
$$\hat{\beta} = .0104$$

After 48 hours of burn-in the residual life $T_{48\ hours}$ should be mixed exponential with parameters α and $\dfrac{\beta}{1+2880\beta}$. The first graph shows the change in $\tilde{\beta}_f$ as a function of burn-in hours. The second graph shows the change in estimated reliabilities at 20 minutes as a function of burn-in time, where reliability at time 20 minutes is estimated to be

$$\hat{R}(t) = [1 + 20\hat{\beta}_f]^{-\hat{\alpha}}$$

$\hat{\beta}_f$ decreases as burn-in time increases.

Forecast $\tilde{\beta}_f$ as a function of burn-in time

Estimated Burn-in Reliability at 20 minutes
as a function of previous burn-in time
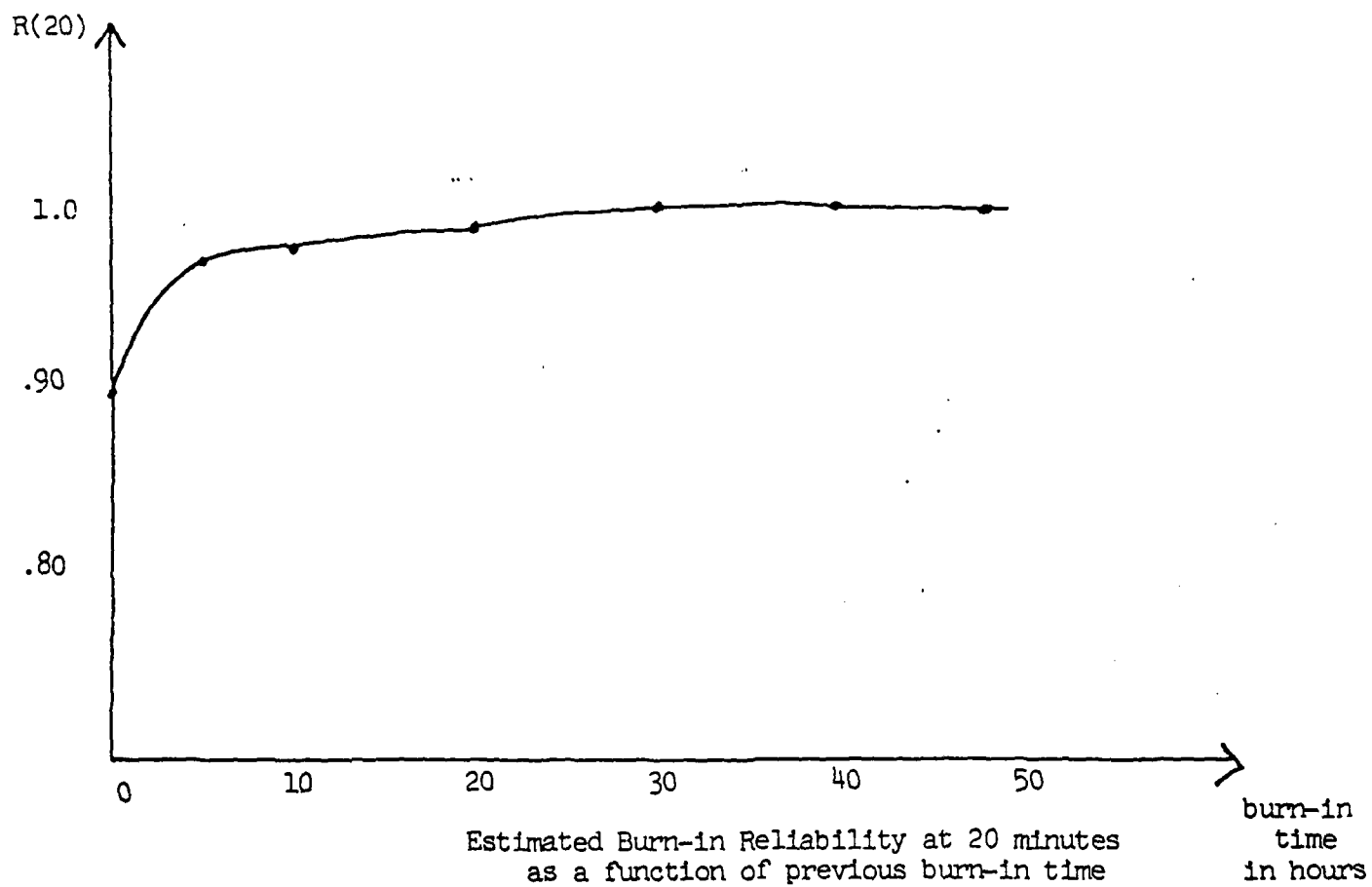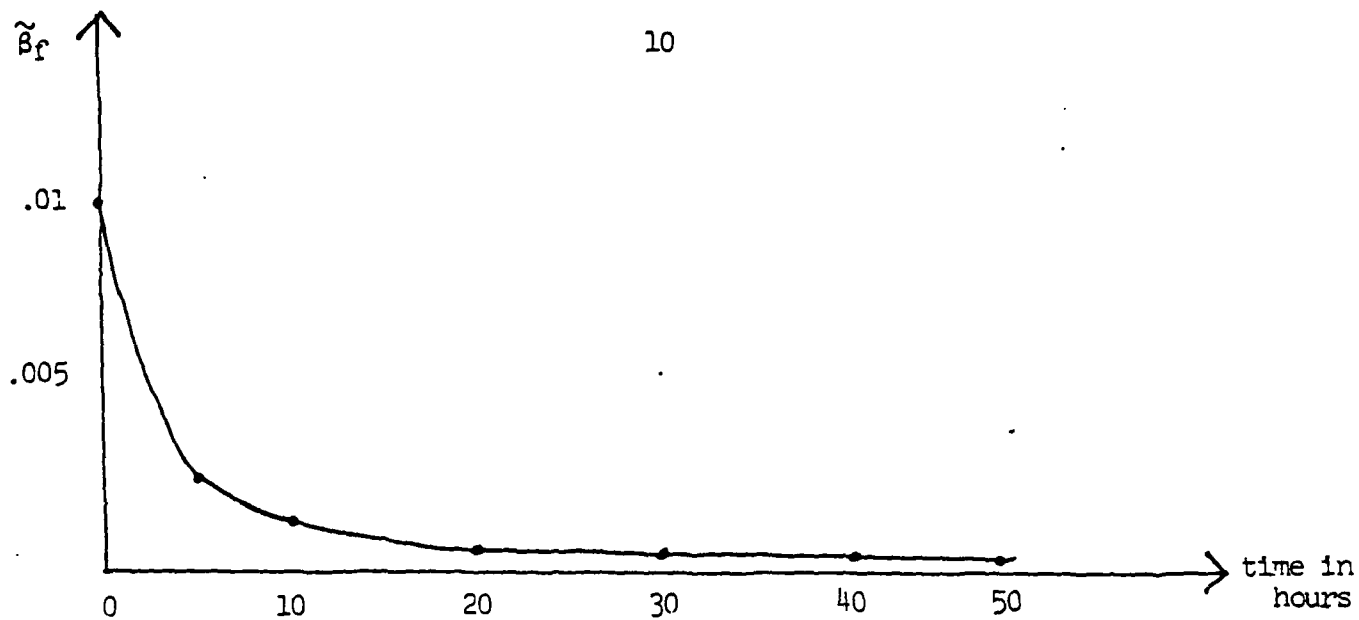
Data consistent with success and failure data was obtained from another test called Pre-Test. We assume that the time to failure T of flight control packages subjected to this type of test environment follows a mixed exponential distribution with shape parameter α and scale parameter β. Using Pre-Test data these parameters were estimated (formulas in Section 5) to be

$$\hat{\alpha} = .5739$$
$$\hat{\beta} = .1106$$

Pre-Test

60 minutes
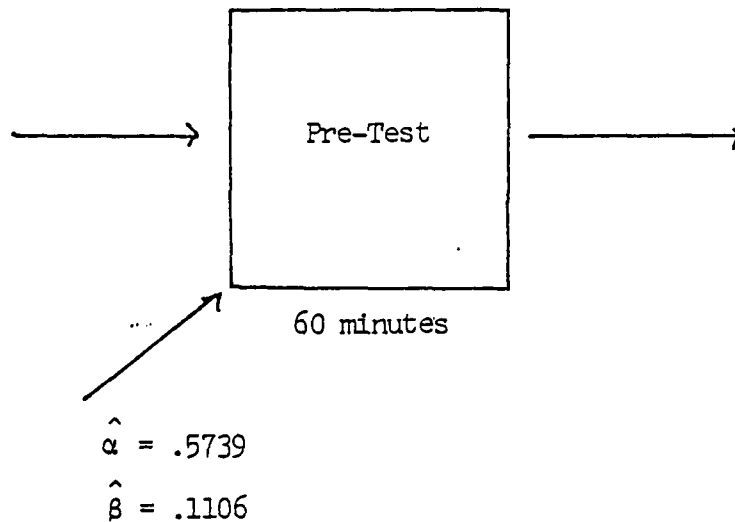
$$\hat{\alpha} = .5739$$
$$\hat{\beta} = .1106$$

After 60 minutes of test the residual life T_{60} should be a mixed exponential with parameters α and $\dfrac{\beta}{1+60\beta}$. We estimate these parameters by

$$\hat{\alpha} = .5739$$
$$\tilde{\beta}_f = \frac{\hat{\beta}}{1+t\hat{\beta}} = \frac{.1106}{1 + 60(.1106)} = .0145$$

$$\tilde{\beta}_f = \frac{\hat{\beta}}{1 + 60\hat{\beta}} = .0145$$

$$\hat{R}_f(1) = .99$$

Pre-Test

t = 60 min
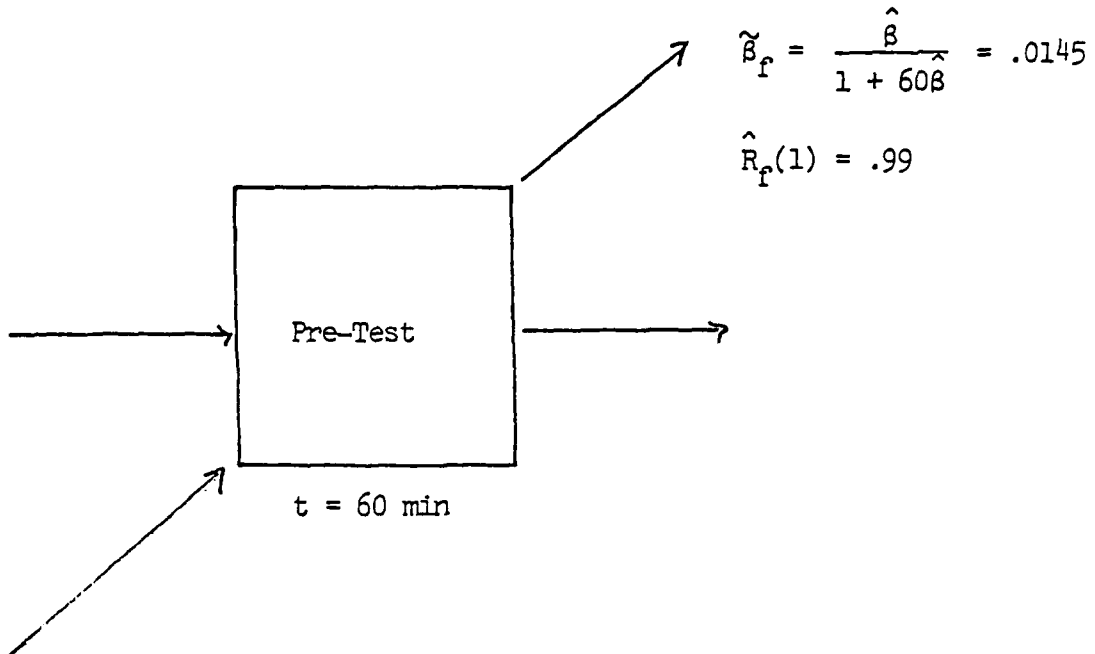
$$\hat{\alpha} = .5739$$

$$\hat{\beta} = .1106$$

$$\hat{R}(1) = .94$$

Since $R(t) = (1 + \beta t)^{-\alpha}$, we estimate reliability at 1 minute for a package which has not gone through Pre-Test to be

$$\hat{R}_{\hat{\beta}}(60) = [1 + (.1106)(1)]^{-.5739} = .94$$

We estimate reliability at 1 minute for a package which has gone through Pre-Test to be

$$\hat{R}_{\hat{\beta}_f}(60) = [1 + (.0145)(1)]^{-.5739} = .99$$

Note that these reliabilities are for Pre-Test environments.

Now consider the entire screen test scheme for Trident flight control packages.

```
┌──────────────┐      ┌──────────────┐      ┌──────────────┐
│  Pre-Test    │      │    No. 1     │      │    Post      │
│              │ ───▶ │   Burn-In    │ ───▶ │  Burn-In     │ ───▶
│ (Job Stack)  │      │              │      │ (Job Stack)  │
└──────────────┘      └──────────────┘      └──────────────┘

     ┌──────────────┐      ┌──────────────┐      ┌──────────────┐      ┌──────────────┐
     │    No. 1     │      │    Post      │      │    Post      │      │    No. 2     │
 ──▶ │    Temp      │ ───▶ │  Temp.Cyc.   │ ───▶ │   Closure    │ ───▶ │    Temp.     │ ───▶
     │    Cyc.      │      │ (Job Stack)  │      │ (Job Stack)  │      │    Cyc.      │
     └──────────────┘      └──────────────┘      └──────────────┘      └──────────────┘

     ┌──────────────┐      ┌──────────────┐      ┌──────────────┐      ┌──────────────┐
     │    Post      │      │    Shock     │      │    Post      │      │   Pressure   │
 ──▶ │  Temp. Cyc.  │ ───▶ │  Vibration   │ ───▶ │    S.&V.     │ ───▶ │   Altitude   │ ───▶
     │ (Job Stack)  │      │              │      │ (Job Stack)  │      │              │
     └──────────────┘      └──────────────┘      └──────────────┘      └──────────────┘

     ┌──────────────┐      ┌──────────────┐      ┌──────────────┐      ┌──────────────┐
     │    Post      │      │    No. 2     │      │    Post      │      │    No. 2     │
 ──▶ │    P.&A.     │ ───▶ │   Burn-In    │ ───▶ │   Burn-In    │ ───▶ │  Continuity  │ ───▶
     │ (Job Stack)  │      │              │      │ (Job Stack)  │      │              │
     └──────────────┘      └──────────────┘      └──────────────┘      └──────────────┘

                     ┌──────────────┐
                     │    Final     │
                 ──▶ │     ATP      │
                     │ (Job Stack)  │
                     └──────────────┘
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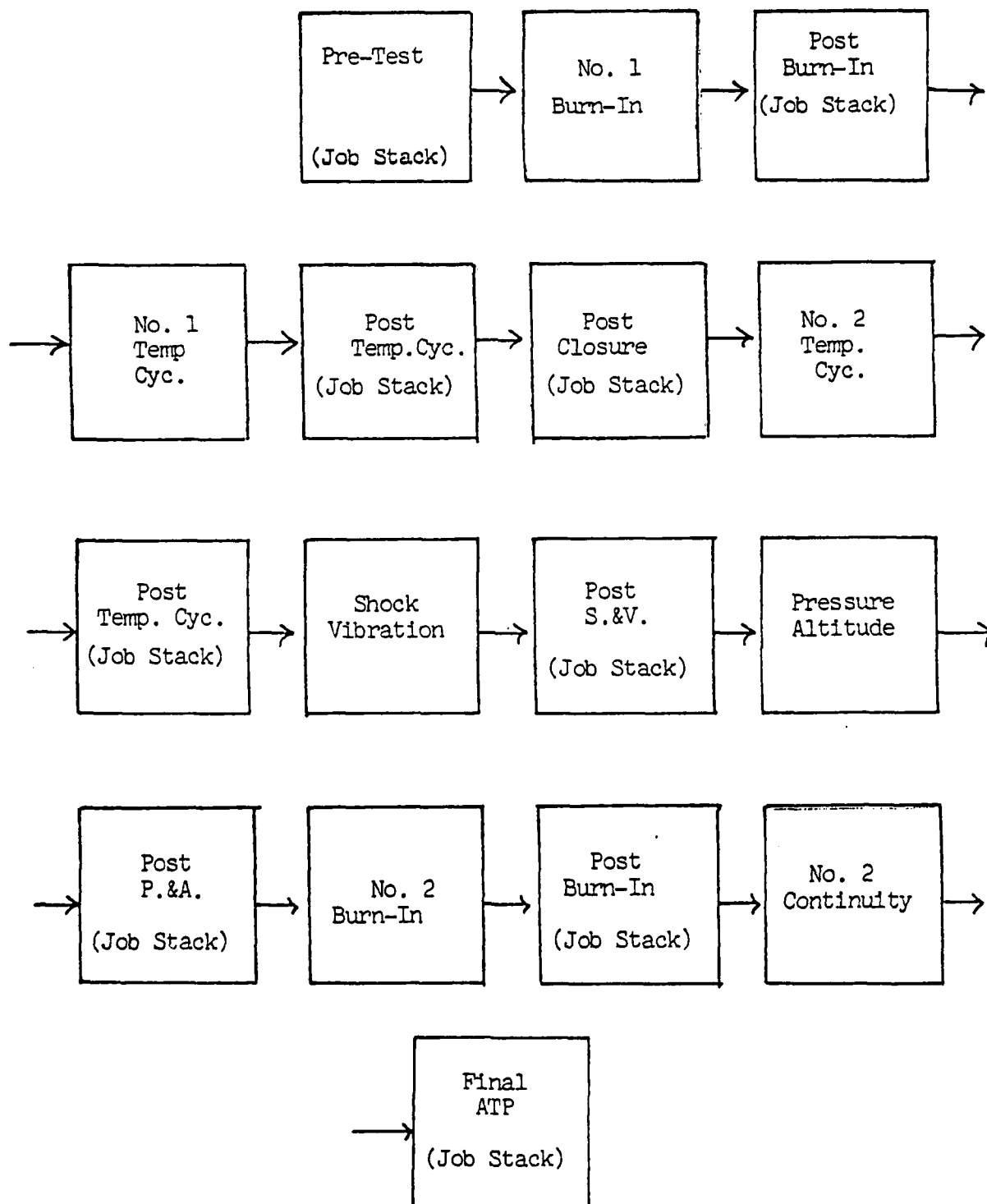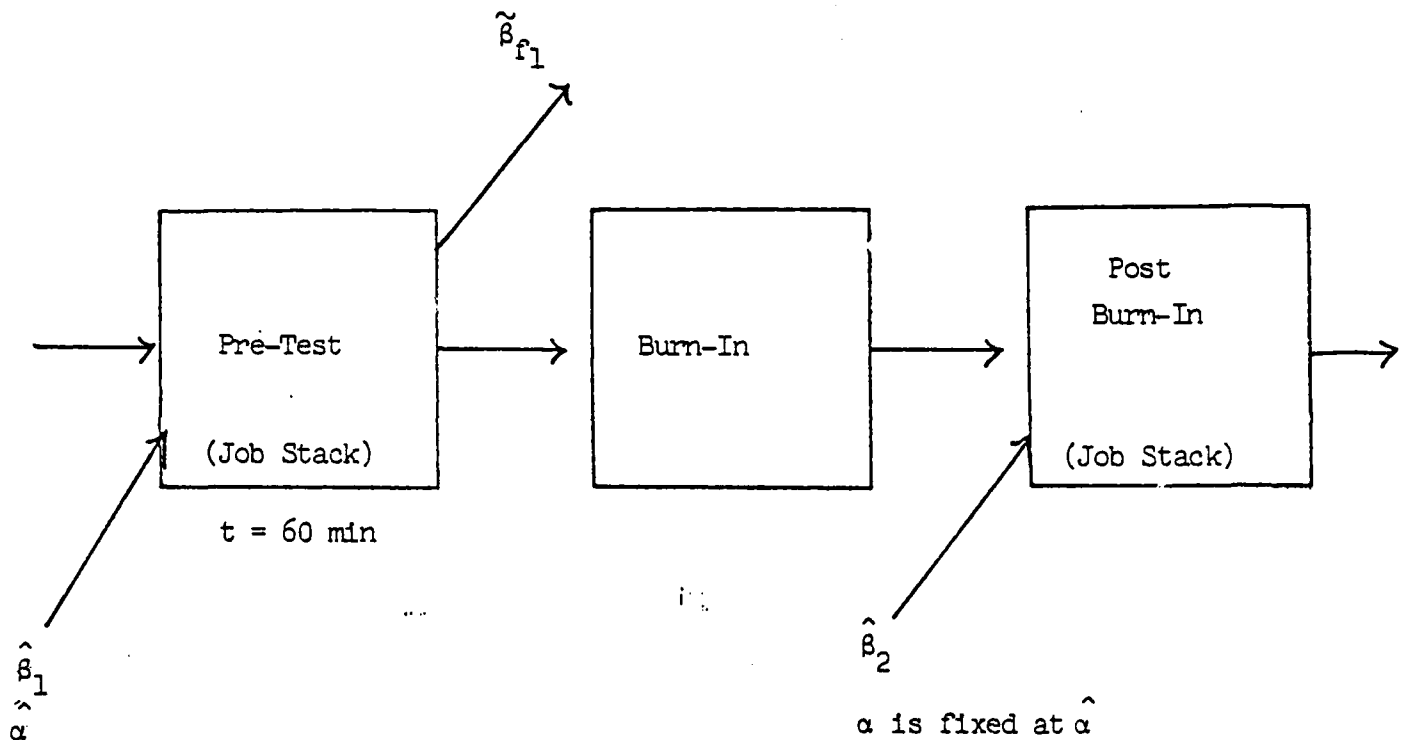
Note: All Job Stack tests are the same.

Effects of various environmental or burn-in tests (in the sense of reliability gain) can be estimated by comparing reliability estimates forecast at the end of the Job Stack which proceeds the environment to reliability estimates for the Job Stack which follows the environment. For example

$$\tilde{\beta}_{f_1}$$

Pre-Test (Job Stack)		Burn-In		Post Burn-In (Job Stack)

t = 60 min

$$\hat{\beta}_1$$
$$\hat{\alpha}$$

$$\hat{\beta}_2$$

α is fixed at $\hat{\alpha}$

If burn-in is effective then we would expect that

$$\hat{\beta}_2 \; < \; \tilde{\beta}_{f_1}$$

5. Estimation of Parameters with Censored Data

Let us assume throughout this section that we are given $t_1, \ldots, t_k$ as observed times of failure while $t_{k+1}, \ldots, t_n$ are observed alive-times both obtained from a mixed exponential (α, β) life distribution with $1 \leq k \leq n$. We define two functions for $x > 0$.

$$S_1(x) = \sum_{i=1}^{n} \ln(1+t_i x) \ , \quad S_2(x) = \frac{1}{k} \sum_{i=1}^{k} (1+t_i x)^{-1}$$

A result on the maximum likelihood estimation (m.l.e.) of the unknown parameters is now given which utilizes data of this type.

Theorem: Under the assumptions and conditions given

(i) When $\beta > 0$ is known, there exists a unique

m.l.e. of α, say $\hat{\alpha}$, given explicitly by

$$\hat{\alpha} = k/S_1(\beta) \ .$$

(ii) When $\alpha > 0$ is known, there exists a unique
m.l.e. of β, say $\hat{\beta}$, given explicitly by

$$\hat{\beta} = A^{-1}(0)$$

where A is the monotone decreasing function
defined by

$$A(x) = kS_2(x) - \alpha x S_1'(x) \quad \text{for} \quad x > 0$$

with primes denoting derivatives.

(iii)   When α, β are both unknown, the m.l.e. of β, say $\hat{\beta}$, is given implicitly, when it exists positively and finitely, by

$$\hat{\beta} = B^{-1}(0)$$

where B is the function defined by

$$B(x) = \frac{S_2(x)}{x} - \frac{S_1'(x)}{S_1(x)} \qquad \text{for } x > 0$$

and the m.l.e. of α, say $\hat{\alpha}$, is given explicitly by

$$\hat{\alpha} = k/S_1(\hat{\beta}) .$$

Theorem: The inequality for $1 \leq k \leq n$

$$\frac{2}{k} \sum_{i=1}^{k} t_i \sum_{j=1}^{n} t_j < \sum_{j=1}^{n} t_j^2 \qquad (4)$$

is a sufficient condition which a (censored) sample from a mixed exponential (α, β) distribution must satisfy in order that maximum likelihood estimators of both parameters exist both positively and finitely.

6. Computational Considerations

The question which now arises is: what kinds of samples will satisfy condition (4)? If k = n we see (4) is equivalent with

$$\left(\frac{1}{n} \sum_1^n t_i\right)^2 < \frac{1}{n} \sum_1^n t_i^2 - \left(\frac{1}{n} \sum_1^n t_i\right)^2$$

from which we have the

Remark: A complete sample of failure times will satisfy (4) if the sample standard deviation exceeds the sample mean.

It can be shown that if T has a mixed exponential (α, β) distribution then

$$E[T] = [\beta(\alpha-1)]^{-1} \quad \text{for} \quad \alpha > 1$$

$$Var[T] = \alpha[\beta^2(\alpha-1)^2(\alpha-2)^2]^{-1} \quad \text{for} \quad \alpha > 2$$

Thus the standard deviation does exceed the mean for those values of the parameters where the mean E[T] and the variance, V[T], exist.

Remark: A sample with k < n failure times and the remaining n-k observations truncated at t_0 will satisfy (4) if

$$t_0 > n_1\left[1 + \sqrt{\frac{2k}{n-k} + 1}\right] \approx n_1 \frac{2n-k}{n-k} \quad \text{for} \quad n \quad \text{large}$$

where $n_1 = (t_1 + \ldots + t_k)/k$ is the average failure time.

In the calculation of $\hat{\beta}$ the equation, $C(\beta) = 0$, must be solved where $C(\beta) = \beta S_1'(\beta)S_2(\beta)$ or

$$C(\beta) = \sum_{j=1}^n \frac{t_j\beta}{1+t_j\beta} - \sum_{j=1}^n \ln(1+t_j\beta) \sum_{i=1}^k \frac{1}{1+t_i\beta}$$

where $t_1, \ldots, t_k$ are failure times and $t_{k+1}, \ldots, t_n$ are censored life

times. We introduce notation for the sample moments as follows:

$$n_r = \frac{1}{k} \sum_{i=1}^{k} t_i{}^r \;, \quad \zeta_r = \frac{1}{n} \sum_{j=1}^{n} t_j{}^r \quad \text{for} \quad r = 1,2,3,\ldots, \quad (5)$$

then using the two expansions, valid for $|x| < 1$,

$$\ell n(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots \;, \quad \frac{1}{1+x} = 1 - x + x^2 - \ldots$$

and substituting into C and simplifying we find, upon neglecting

terms of third order in β , that

$$(1+n_1\beta+n_2\beta)(\zeta_1+\zeta_2\frac{\beta}{2}+\zeta_3\frac{\beta^2}{3}) - [\zeta_1 - \zeta_2\beta + \zeta_3\beta^2] = 0$$

Multiplying the first two together and collecting terms yields

$$(\frac{\zeta_2}{2} - n_1\zeta_1)\beta - (\zeta_3 - n_2\zeta_1 - \frac{n_1\zeta_2}{2})\beta^2 = 0$$

We now notice that the condition equation (4), can be written in the

notation of (5) as $\zeta_2 > 2n_1\zeta_1$.

Thus our computational procedure to decide upon the parametric

representation of the distribution governing the observations which have

been obtained is contained in the following.

Algorithm: Given $t_1, \ldots, t_k$ as failure times and $t_{k+1}, \ldots, t_n$ as

censored times from a mixed exponential (α, β) distribution

(1) Compute the sample moments $n_1, n_2, \zeta_1, \zeta_2, \zeta_3$.

(ii) If $\zeta_2 < 2\eta_1\zeta_1$, assume observations from a constnat failure rate distribution and estimate λ by

$$\hat{\lambda} = \frac{k}{n\zeta_1} \ .$$

(iii) If $\zeta_2 > 2\eta_1\zeta_1$, assume observations are from a mixed exponential distribution and compute

$$\beta_0 = \frac{\zeta_2 - 2\eta_1\zeta_1}{2\zeta_3 - 2\eta_2\zeta_1 - \eta_1\zeta_2}$$

then use the Newton-Raphson interation procedure, namely for $n = 0, 1, 2, \ldots$.

$$\beta_{n+1} = \beta_n - \frac{C(\beta_n)}{C'(\beta_n)}, \quad \hat{\beta} = \lim_{n \to \infty} \beta_n \ , \text{ and}$$

$$\hat{\alpha} = \frac{k}{\sum_{j=1}^{n} \ln(1 + t_j\hat{\beta})} \ .$$

Practical experience indicates that the iteration converges very rapidly. Since the functions are very simple a small programmable electronic calculator, such as the HP-65, can be used to obtain these estimates. Programs for the HP-65 and HP-97 are available from the authors.

7. Conclusion

If a component has a life distribution with an increasing
failure rate, the information necessary to estimate its parameters
must contain failure times. In practice this means that virtually
no observed failures, within a fleet of operational components, provide
little information with which to assess reliability.

If a component has a constant failure rate then both failure
times and alive times contribute equally to its estimation. The
preceeding study suggests that if a component has a life distribution
with decreasing failure rate it is the alive times within the data
which contribute principally to the estimation of the parameters.

The problem of obtaining the usual sampling distributions of
the maximum likelihood estimators of the parameters for the decreasing
failure rate model studied seems to be difficult because the estimates
are only implicitly defined. We have shown, however, that when they
exist the MLE's for α and β, based on type I or on random censoring,
are asymptotically normally distributed. We have also shown that the
distribution function estimated using the joint MLE's of the parameters
is surprisingly closer to the true distribution for regions of interest
in reliability theory, than is the estimated distribution function
using a BLUE-k estimate for the scale parameter and a known shape parameter.

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